Correlation Study of Distances Calculated Using Absorption Line Identification and Cosmological Parameters from *Plank* Data Release

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Abstract

The vast distance between objects in the universe can be directly calculated by observing it's spectra. The red-shift of the object due to the Hubble flow helps to find the relativistic distance (D_S) of the object. The cosmological parameters Ω_M, Ω_k and Ω_Λ could be used to calculate the co-moving distance (D_C), the angular distance (D_A) and the luminosity distance (D_L). The focus of this report was to take spectra of four galaxies and calculate the relativistic distance by identifying the location of the $H\alpha$ and $H\beta$ lines. D_C, D_A and D_L were calculated with cosmological parameters with $\Omega_\Lambda = 0.6847 \pm 0.0073, \Omega_M = 0.3153 \pm 0.0073, \Omega_k = 0.001 \pm 0.002$ and the Hubble constant $H_0 = 72$ (km/s)/MPc

Keywords: Redshift, Cosmological Parameter, Comoving Distance, Hubble Distance, Relativistic Distance

1 Introduction

In 1929 Edwin Hubble observed the nearby galaxies and concluded that the galaxies are receding away from us, thus making it a proof for the expanding universe.[1] If the galaxy is at rest with respect to the observer then we will see a rest spectrum. The spectrum will contain different absorption lines according to what the galaxy is made of. If there is a relative velocity between the galaxy and the observer, then we will see a shift of the absorption lines in the spectrum. We can measure the shift of these lines and determine the distance of the galaxy from the observer using Doppler's formalism.

In this paper, we have derived this distance directly using spectra and then compared this distance with the value found in the literature. The co-moving distance, the angular distance and luminosity distance were derived using cosmological parameters. The formulation of these distances is discussed in the Methodology section. A detailed discussion on making a choice while selecting a dataset can be found in the Data section. We then discussed about our result and concluded our findings in the Results & Discussion section.

2 Methodology

In particular, the $H\alpha$ line at 6563 Å and $H\beta$ line at 4861 Å can be detected easily in a galaxy spectrum, as hydrogen is abundant in the observable universe. The wavelengths can be determined using the Rydberg equation. $H\alpha$ line wavelength corresponds to the wavelength of the electron that jumps from the third orbit to the second orbit. $H\beta$ line wavelength corresponds to the wavelength of the electron that jumps from the fourth orbit to the second orbit. [2]

$$\frac{1}{\lambda_{\text{rest}}} = \frac{1}{R_H}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Due to the relative velocity of the galaxies with respect of the observer using Doppler effect, the red-shift can be derived as:

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{src}}}{\lambda_{\text{src}}}$$

The λ_{obs} is determined from observing the spectra of the galaxy. Once the red shift value of the galaxy is determined we can calculate the receding velocity of the galaxy using $v = cz$ and then using the Hubble's formula to calculate distance of the galaxy.

$$D_S = \frac{v}{H_0}$$

The co-moving distance of a galaxy can be determined using cosmological parameters that shape the geometry of our universe. The Friedmann equation is used to determine the geometry and is expressed as[3]:

$$H_0^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

In this equation, a is the scale factor, H_0 is the Hubble constant, G is known as the Newton's gravitational constant and Λ is the cosmological constant. Dividing the equation by H_0^2 we get the three cosmological parameters[4].

$$\Omega_m = \frac{8\pi G\rho}{3H_0^2}; \quad \Omega_k = -\frac{kc^2}{a^2 H_0^2}; \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$$

In this formulation, $\Omega_M, \Omega_k, \Omega_\Lambda$ are respectively the matter density parameter, spatial curvature density parameter and the dark energy density parameter. The theoretical relationship between these parameters put together becomes:

$$\Omega_M + \Omega_k + \Omega_\Lambda = 1$$

In Plank 2018 Data release [5] we get the observational values of these cosmological parameters.

Parameter	**TT,TE,EE+lowE+lensing 68% limits**
Ω_Λ	0.6847 ± 0.0073
Ω_M	0.3153 ± 0.0073
Ω_k	0.001 ± 0.002

Using these parameters the co-moving distance can be calculated using the following formula[4]:

$$D_C = D_H \int_0^z \frac{dz}{\sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}}$$

Where, D_H is the Hubble distance that can be defined as the ratio of the speed of light to the Hubble constant. Currently the value of the Hubble distance is 41663.785 Mpc if we take the value of H_0 to be 72km/s/Mpc [4]

And transverse co-moving distance (D_M) which is the distance between two points in the sky (side by side) is $D_C = D_M$ when the dark energy density parameter Ω_k is equal to zero meaning the geometry of the universe is flat. In the telescope, only the angular size of an object is received. But it is important to get the actual size to make the calculations accurate. For that, the angular diameter distance(D_A) is used. D_{Lambda} and D_M 's mathematical relationship:

$$D_A = \frac{D_M}{1+z}$$

It can be seen that the value of $D_A > D_M$ as the red shift value is always a positive number. To make the calculation easy and accurate for the purpose of determining the distance, there is a mathematical relationship between the transverse co-moving distance and luminosity distance to determine the luminosity distance:

$$D_L = (1+z)D_M$$

In astrophysics, it is known that when the universe expands the light waves are also stretched which makes the observed wavelength to increase by a factor of $1+z$.

3 Data

The datasets for this project were obtained from the NED Extragalactic Database maintained by NASA. [6] In order to detect the red-shifted H and H lines we concentrated our attention to the range of 4000 Å to 7000 Å. This range was chosen because it has been determined that the $H\alpha$ line with a rest frame wavelength of 6562.8 Å and the $H\beta$ line with an interval of 4861 Å are located within the specified limits.

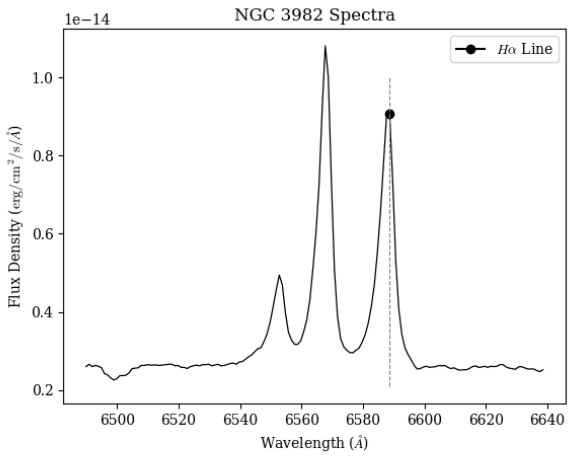

Figure 1. $H\alpha$ line red shifted to another wavelength in the spectra

In particular, progressive tolerance was adopted in selecting these data sets to enhance the quality of the analysis through a minimal step size between each set of wavelengths used. The raw data sets consist of data related to columns of the wavelength (Å), intensity (erg/cm^2/s/Å) and uncertainty (erg/cm^2/s/Å) respectively.

Algorithm 1: Measure Redshift from Spectral Data

```
Data: spectra (contains wavelength and intensity
      values)
Result: redshift (calculated value)
while not at end of spectra do
    observed_wavelength ← rest_wavelength ;
    if observed_wavelength is valid then
        redshift ← (observed_wavelength -
          rest_wavelength) / rest_wavelength ;
        return redshift ;
    end
    else
        print("Invalid range!") ;
    end
end
```

For the purpose of this study, these uncertainty values were not needed and have therefore been omitted from the data set. This cleaning was done with the use of Microsoft Excel. Furthermore, all occurrences of `NaN` were removed.

The $H\alpha$ line (or $H\beta$ line) was identified in the spectra as can be seen in Figure 1. The red shift value (z) was calculated with the formulae stated previously.

4 Results & Discussion

During data analysis, an unusual observation was made in the data set for the object ID 3C 273, where intensity values were significantly higher compared to other objects. Upon investigation, it was determined that this discrepancy was due to the nature of a special type of object called quasars, which are among the most luminous objects in the universe.

Quasars, like 3C 273, are powered by the accretion of matter onto supermassive black holes, resulting in extremely high energy output across the electromagnetic spectrum. 3C 273, in particular, is known for its exceptional brightness, despite its considerable distance from Earth. The combination of this intrinsic luminosity and the small units used for flux density measurements (erg/cm^2/s/Å) explains the notably high intensity values observed.

An overview of the datasets and their relativistic distances can be found in Table 1. Comparison between the four types of distances of each object can be found as a bar chart in Figure 2. The effect of the red shift can be clearly seen in the distance.

Acknowledgement

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Code and Data Availability

The codes used to analyze the distances used in this paper, are publicly available at Astronomy Projects for Amateurs - GitHub Repository The data sets used can also be found in the stated repository.

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Table 1. Dataset Overview with Relativistic Distance

Object ID	Dataset Name	Redshift Value (z)	Relativistic Distance from Calculation (MPc)	Relativistic Distance from Observation (MPc)	Error in per cent
3C 273	`3C_273_S_B_bcc2009_NED`	0.158339	658.11	705.23 [7]	6.68
NGC 4051	`NGC_4051_S_R_hfs1995_NED`	0.002336	12.82	13.64 [8]	6.04
NGC 3982	`NGC_3982_S_R_hfs1995_NED`	0.003741	16.41	19.04 [9]	13.24
NGC 7469	`NGC_7469_Opt_mk2006_NED`	0.016268	69.06	66.47 [10]	3.90

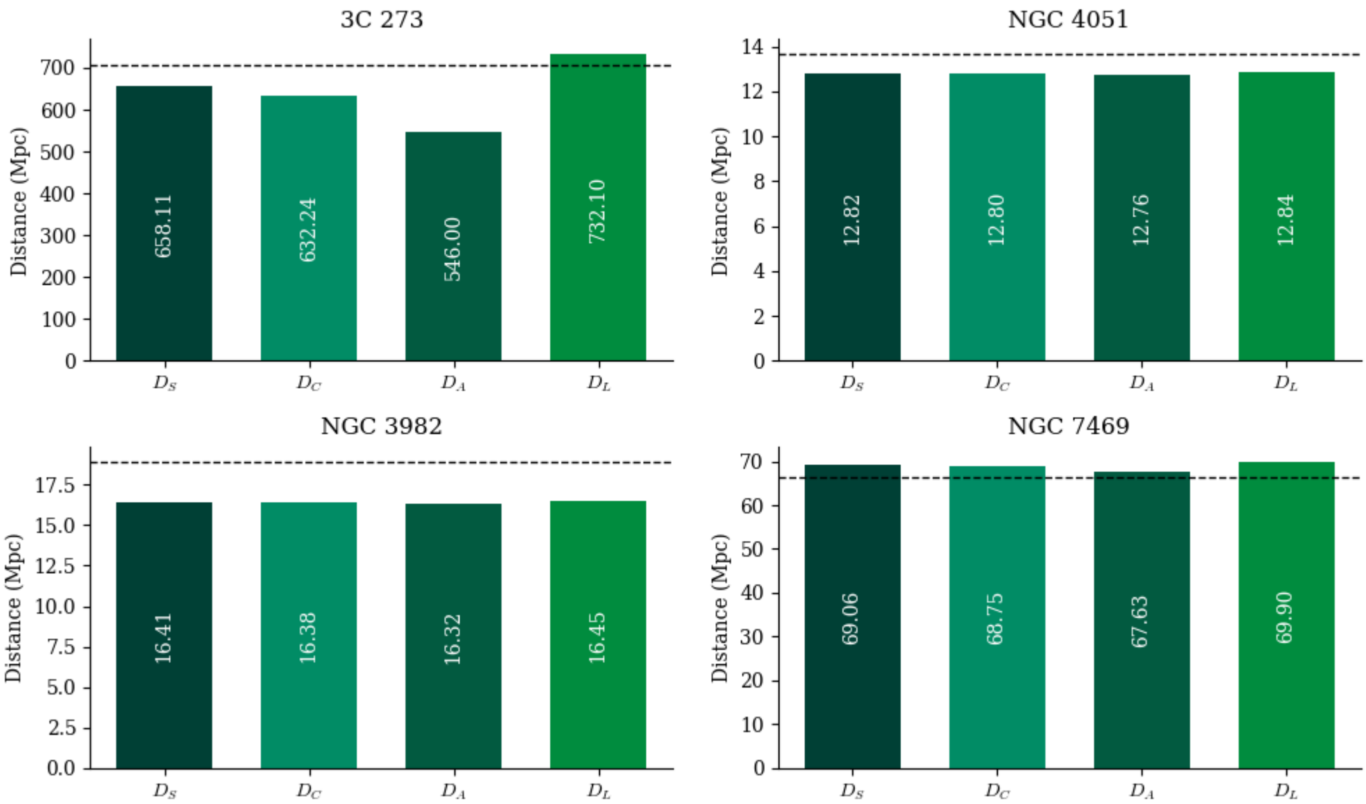

Figure 2. Overview of different types of distance comparison of four objects